SCIENCE

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THE HUMAN WORTH OF RIGOROUS THINKING[1]

But in the strong recess of Harmony,
Established firm abides the rounded Sphere.
—Empedocles.

AMONG the agencies that ameliorate life, what is the rôle of rigorous thinking? What is the rôle of the spirit that aspires always to logical righteousness, seeking "to frame a world according to a rule of divine perfection"?

Evidently that question is not one for adequate handling in an hour's address by an ordinary student of mathematics. Rather is it a subject for a long series of lectures by a learned professor of the history of civilization. Indeed so vast is the subject that even an ordinary student of mathematics can detect some of the more obvious tasks such a philosophic historian would have to perform and a few of the difficulties he would doubtless encounter. It may be worth while to mention some of them.

Certainly one of the tasks, and probably one of the difficulties also, would be that of securing an audience—an audience, I mean, capable of understanding the lectures, for is not a genuine auditor a listener who understands? To understand the lectures it would seem to be necessary to know what that is which the lectures are about—that is, it would be necessary to know what is meant by rigorous thinking. To know this, however, one must either have consciously done some rigorous thinking or else, at the very least, have examined some specimens

[1] An address delivered before the Mathematical Colloquium of Columbia University, October 13, 1913.

of it pretty carefully, just as, in order to know what good art is, it is, in general, essential either to have produced good art or to have attentively examined some specimens of it, or to have done both of these things. Here, then, at the outset our historian would meet a serious difficulty, unless his audience chanced to be one of mathematicians, which is (unfortunately) not likely, inasmuch as the great majority of mathematicians are so exclusively interested in mathematical study or teaching or research as to be but little concerned with the philosophical question of the human worth of their science. It is, therefore, easy to see how our lecturer would have to begin.

Ladies and gentlemen, we have met, he would say, to open a course of lectures dealing with the rôle of rigorous thinking in the history of civilization. In order that the course may be profitable to you, in order that it may be a course in ideas and not merely or mainly a verbal course, it is essential that you should know what rigorous thinking is and what it is not. Even I, your speaker, though a historian, might reasonably be held to the obligation of knowing that.

It is reasonable, ladies and gentlemen, it is reasonable to assume, he would say, that in the course of your education you neglected mathematics, and it is, therefore, probable or indeed quite certain that, notwithstanding your many accomplishments, you do not quite know, or rather, perhaps I should say, you are very far from knowing, what rigorous thinking is or what it is not. Of course, as you know, it is, generally speaking, much easier to tell what a thing is not than to tell what it is, and I might, he would say, I might proceed by way of a preliminary to indicate roughly what rigorous thinking is not. Thus I might explain that rigorous thinking, though much of it has been done in the world, and though it has produced a large literature, is nevertheless a relatively rare phenomenon. I might point out that a vast majority of mankind, a vast majority of educated men and women, have not been disciplined to think rigorously even those things that are most available for such thinking. I might point out that, on the other hand, most of the ideas with which men and women have constantly to deal are as yet too nebulous and vague, too little advanced in the course of their evolution, too little refined and defined, to be available for concatenative thinking and rigorous discourse. I should have to say, he would add, that, on these accounts, most of the thinking done in the world on a given day, whether done by men in the street or by farmers or factory-hands or merchants or administrators or physicians or lawyers or jurists or statesmen or philosophers or men of letters or students of natural science or even mathematicians (when not strictly employed in their own subject), comes far short of the demands and standards of rigorous thinking.

I might go on to caution you, our speaker would say, against the current fallacy, recently advanced by eloquent writers to the dignity of a philosophical tenet, of regarding what is called successful action as the touchstone of rigorous thinking. For you should know that much of what passes in the world for successful action proceeds from impulse or instinct and not from thinking of any kind; you should know that no action under the control of non-rigorous thinking can be strictly successful except by the favor of chance or through accidental compensation of errors; you should know that most of what passes for successful action, most of what the world applauds and even commemorates as successful action, so far from being really

successful, varies from partial failure to failure that, if not total, would at all events be fatal in any universe that had the economic decency to forbid, under pain of death, the unlimited wasting of its resources. The dominant animal of such a universe would be in fact a superman. In our world the natural resources of life are superabundant, and man is poor in reason because he has been the prodigal son of a too opulent mother. But, ladies and gentlemen, our speaker will conclude, you will know better what rigorous thinking is not when once you have learned what it is. This, however, can not well be learned in a course of lectures in which that knowledge is presumed. I have, therefore, to adjourn this course until such time as you shall have gained that knowledge. It can not be gained by reading about it or hearing about it. The easiest way, for some persons the only way, to gain it is to examine with exceeding patience and care some specimens, at least one specimen, of the literature in which rigorous thinking is embodied. Such a specimen, he could say, is Dr. Thomas L. Heath's magnificent edition of Euclid where an excellent translation of the "Elements" from the definitive text of Heiberg is set in the composite light of critical commentary from Aristotle down to the keenest logical microscopists and histologists of our own day. If you think Euclid too ancient, and too stale even when seasoned with the wit of more than two thousand years of the acutest criticism, you may find a shorter and possibly a fresher way by examining minutely such a work as Veronese's "Grundzüge der Geometrie" or Hilbert's famous "Foundations of Geometry" or the late Pieri's "Della Geometria elementare come sistemi ipotetico-deduttivo." In works of this kind, of which the growing number is rather large, and not elsewhere, you will find, in its nakedness, purity and spirit, what you have neglected and what you need. You will note that in the beginning of such a work there is found a system of assumptions or postulates, discovered the Lord only and a few men of genius know where or how, selected perhaps with reference to simplicity and clearness, certainly selected with respect to their compatibility and independence, and, it may be, with respect also to categoricity. You will not fail to observe with the utmost minuteness, and from every possible angle, how it is that upon these postulates as a basis there is built up by a kind of divine masonry, little step by step, a stately structure of ideas, an imposing edifice of rigorous thought, a towering architecture of doctrine that is at once beautiful, austere, sublime and eternal. Ladies and gentlemen, our speaker will say, to accomplish that examination will require twelve months of pretty assiduous application. The next lecture of this course will be given one year from date.

On resuming the course what will our philosopher and historian proceed to say? He will begin to say what, if he says it concisely, will make up a very large volume. Room is lacking here, even if competence were not, for so much as an adequate outline of the matter. It is possible, however, to draw with confidence a few of the larger lines that would have to enter such a sketch.

What is it that our speaker will be obliged to deal with first? I do not mean obliged logically or obliged by an orderly development of his subject. I mean obliged by the expectation of his hearers. Every one can answer that question. For presumably the audience represents the spirit of the times, and this age is, at least to a superficial observer, an age of engineering. Now, what is engineering? Well, the charter of the Institution of Civil

Engineers tells us that engineering is the "art of directing the great sources of power in Nature for the use and convenience of man." By Nature here must be meant external or physical nature, for, if internal nature were also meant, *every* good form of activity would be a species of engineering, and may be it is such, but that is a claim which even engineers would hardly make and poets would certainly deny. Use and convenience—these are the key-bearing words. It is perfectly evident that our lecturer will have to deal first of all with what the world would call the "utility" of rigorous thinking, that is to say, with the applications of mathematics and especially with its applications to problems of engineering. If he really knows profoundly what mathematics is, he will not wish to begin with applications or even to make applications a major theme of his discourse, but he must, and he will do so uncomplainingly as a concession to the external-mindedness of his time and his audience. He will not only desire to show his audience applications of mathematics to engineering, but, being a historian of civilization, he will especially desire to show them the development of such applications from the earliest times, from the building of pyramids and the mensuration of land in ancient Egypt down to such splendid modern achievements as the designing and construction of an Eads bridge, an ocean *Imperator* or a Panama canal. The story will be long and difficult, but it will edify. The audience will be amazed at the truth if they understand. If they do not understand the truth fully, our speaker must at all events contrive that they shall see it in glimmers and gleams and, above all, that they shall acquire a feeling for it. They must be led to some acquaintance with the great engineering works of the world, past and present; they must be given an intelligent conception of the immeasurable contribution such works have made to the comfort, convenience and power of man; and especially must they be convinced of the fact that not only would the greatest of such achievements have been, except for mathematics, utterly impossible, but that such of the lesser ones as could have been wrought without mathematical help could not have been thus accomplished without wicked and pathetic waste both of material resources and of human toil. In respect to this latter point, the relation of mathematics to practical economy in large affairs, our speaker will no doubt invite his hearers to read and reflect upon the ancient work of Frontinus on the "Water Supply of the City of Rome" in order that thus they may gain a vivid idea of the fact that the most *practical* people of history, despising mathematics and the finer intellectualizations of the Greeks, were unable to accomplish their own great engineering feats except through appalling waste of materials and men. Our lecturer will not be content, however, with showing the service of mathematics in the prevention of waste; he will show that it is indispensable to the productivity and trade of the modern world. Before quitting this division of his subject he will have demonstrated that, if all the contributions which mathematics has made, and which nothing else could make, to navigation, to the building of railways, to the construction of ships, to the subjugation of wind and wave, electricity and heat, and many other forms and manifestations of energy, he will have demonstrated, I say, and the audience will finally understand, that, if all these contributions were suddenly withdrawn, the life and body of industry and commerce would suddenly collapse as by a paralytic stroke, the now splendid outer tokens of material civilization would perish, and the face of our

planet would quickly assume the aspect of a ruined and bankrupt world.

As our lecturer has been constrained by circumstances to back into his subject, as he has, that is, been compelled to treat first of the service that mathematics has rendered engineering, he will probably next speak of the applications of mathematics to the so-called natural sciences—the more properly called experimental sciences—of physics, chemistry, biology, economics, psychology, and the like. Here his task, if it is to be, as it ought to be, expository as well as narrative, will be exceedingly hard. For how can he weave into his narrative an intelligible exposition of Newton's "Principia," Laplace's "Mécanique Céleste," Lagrange's "Mécanique Analytique," Gauss's "Theoria Motus Corporum Cœlestium," Fourier's "Théorie de la Chaleur," Maxwell's "Electricity and Magnetism," not to mention scores of other equally difficult and hardly less important works of a mathematical-physical character? Even if our speaker knew it all, which no man can, he could not tell it all intelligibly to his hearers. These will have to be content with a rather general and superficial view, with a somewhat vague intuition of the truth, with fragmentary and analogical insights gained through settings-forth of great things by small; and they will have to help themselves and their speaker, too, by much pertinent reading. No doubt the speaker will require his hearers, in order that they may thus gain a tolerable perspective, to read well not only the two volumes of the magnificent work of John Theodore Merz dealing with the history of European thought in the nineteenth century, but also many selected portions of the kindred literature there cited in richest profusion. The work treats mainly of natural science, but it deals with it philosophically, under the larger aspect, that is, of science regarded as thought. By the help of such literature in the hands of his auditors, our lecturer will be able to give them a pretty vivid sense of the great and increasing rôle of mathematics in suggesting, formulating and solving problems in all branches of natural science. Whether it be with "the astronomical view of nature" that he is dealing, or "the atomic view" or "the mechanical view" or "the physical view" or "the morphological view" or "the genetic view" or "the vitalistic view" or "the psychophysical view" or "the statistical view," in every case, in all these great attempts of reason to create or to find a cosmos amid the chaos of the external world, the presence of mathematics and its manifold service, both as instrument and as norm, illustrate and confirm the Kantian and Riemannian conception of natural science as "the attempt to understand nature by means of exact concepts."

In connection with this division of his subject, our speaker will find it easy to enter more deeply into the spirit and marrow of it. He will be able to make it clear that there is a sense, a just and important sense, in which all thinkers and especially students of natural science, though their thinking is for the most part not rigorous, are yet themselves contributors to mathematics. I do not refer to the powerful stimulation of mathematics by natural science in furnishing it with many of its problems and in constantly seeking its aid. What I mean is that all thinkers and especially students of natural science are engaged, both consciously and unconsciously, both intentionally and unintentionally, in the mathematicization of concepts—that is to say, in so transforming and refining concepts as to fit them finally for the amenities of logic and the austerities of rigorous thinking. We are dealing here, our speaker

will say, with a process transcending conscious design. We are dealing with a process deep in the nature and being of the psychic world. Like a child, an idea, once it is born, once it has come into the realm of spiritual light, possibly long before such birth, enters upon a career, a career, however, that, unlike the child's, seems to be immortal. In most cases and probably in all, an idea, on entering the world of consciousness, is vague, nebulous, formless, not at once betraying either what it is or what it is destined to become. Ideas, however, are under an impulse and law of amelioration. The path of their upward striving and evolution—often a long and winding way—leads towards precision and perfection of form. The goal is mathematics. Witness, for example, our lecturer will say, the age-long travail and aspiration of the great concept now known as mathematical continuity, a concept whose inner structure is even now known and understood only of mathematicians, though the ancient Greeks helped in moulding its form and though it has long been, if somewhat blindly, yet constantly employed in natural science as when a physicist, for example, or an astronomer uses such numbers as e and π in computation. Witness, again, how that supreme concept of mathematics, the concept of function, has struggled through thousands of years to win at length its present precision of form out of the nebulous sense, which all minds have, of the mere dependence of things on other things. Witness, too, he will say, the mathematical concept of infinity, which prior to a half-century ago was still too vague for logical discourse, though from remotest antiquity the great idea has played a conspicuous rôle, mainly emotional, in theology, philosophy and science. Like examples abound, showing that one of the most impressive and significant phenomena in the life of the psychic world, if we will but discern and contemplate it, is the process by which ideas advance, often slowly indeed but surely, from their initial condition of formlessness and indetermination to the mathematical estate. The chemicization of biology, the physicization of chemistry, the mechanicization of physics, the mathematicization of mechanics, these well-known tendencies and drifts in science do but illustrate on a large scale the ubiquitous process in question.

At length, ladies and gentlemen, our speaker will say, in the light of the last consideration the deeper and larger aspects of our subject are beginning to show themselves and there is dawning upon us a wonderful vision. The nature, function and life of the entire conceptual world seem to come within the circle and scope of our present enterprise. We are beginning to see that to challenge the human worth of mathematics, to challenge the worth of rigorous thinking, is to challenge the worth of all thinking, for now we see that mathematics is but the ideal to which all thinking, by an inevitable process and law of the human spirit, constantly aspires. We see that to challenge the worth of that ideal is to arraign before the bar of values what seems the deepest process and inmost law of the universe of thought. Indeed we see that in defending mathematics we are really defending a cause yet more momentous, the whole cause, namely, of the conceptual procedure of science and the conceptual activity of the human mind, for mathematics is nothing but such conceptual procedure and activity come to its maturity, purity and perfection.

Now, ladies and gentlemen, our lecturer will say, I can not in this course deal explicitly and fully with this larger issue. But, he will say, we are living in a day when that issue has been raised; we happen

to be living in a time when, under the brilliant and effective leadership of such thinkers as Professor Bergson and the late Professor James, the method of concepts, the method of intellect, the method of science, is being powerfully assailed; and, whilst I heartily welcome this attack of criticism as causing scientific men to reflect more deeply on the method of science, as exhibiting more clearly the inherent limitations of the method, and as showing that life is so rich as to have many precious interests and the world much truth beyond the reach of that method, yet I can not refrain, he will say, from attempting to point out rather carefully what seems to me a radical error of the critics, a fundamental error of theirs, in respect to what is the highest function of conception and in respect to what is the real aim and ideal of the life of intellect. For we shall thus be led to a deeper view of our subject proper.

These critics find, as all of us find, that what we call mind or our minds are, in some mysterious way, functionally connected with certain living organisms known as human bodies; they find that these living bodies are constantly immersed in a universe of matter and motion in which they are continually pushed and pulled, heated and cooled, buffeted and jostled about—a universe that, according to James, would, in the absence of concepts, reveal itself as "a big blooming buzzing confusion"—though it is hard to see how such a revelation could happen to any one devoid of the concept "confusion," but let that pass; they find that our minds get into some initial sort of knowing connection with that external blooming confusion through what they call the sensibility of our bodies, yielding all manner of sensations as of weights, pressures, pushes and pulls, of intensities and extensities of brightness, sound, time, colors, space, odors, tastes, and so on; they find that we must, on pain of organic extinction, take some account of these elements of the material world; they find that, as a fact, we human beings constantly deal with these elements through the instrumentality of concepts; they find that the effectiveness of our dealing with the material world is precisely due to our dealing with it conceptually: they infer that, therefore, dealing with matter is exactly what concepts are for, saying with Ostwald, for example, that the goal of natural science, the goal of the conceptual method of mind, "is the domination of nature by man;" not only, our speaker will say, do our critics find that we deal with the material world conceptually, and effectively because conceptually, but they find also that life has interests and the world values not accessible to the conceptual method, and as this method is the method of the intellect, they conclude, not only that the intellect can not grasp life, but that the aim and ideal of intellect is the understanding and subjugation of matter, saying with Professor Bergson "that our intellect is intended to think matter," "that our concepts have been formed on the model of solids," "that the essential function of our intellect . . . is to be a light for our conduct, to make ready for our action on things," that "the intellect is characterized by a natural inability to understand life," that "intellect always behaves as if it were fascinated by the contemplation of inert matter," that "intelligence . . . aims at a practically useful end," that "the intellect is never quite at its ease, . . . except when it is working upon inert matter, more particularly upon solids," and much more to the same effect.

Now, ladies and gentlemen, our speaker will ask, what are we to think of this? What are we to think of this valuation of the science-making method of concepts? What are we to think of the aim and ideal

here ascribed to the intellect and of the station assigned it among the faculties of the human mind? In the first place, he will say, it ought to be evident to the critics themselves, and evident to them even in what they esteem the poor light of intellect, that the above-sketched movement of their minds is a logically unsound movement. They do not indeed contend that, because a living being in order to live must deal with the material world, it must, therefore, do so by means of concepts. The lower animals have taught them better. But neither does it follow that, because certain bipeds in dealing with the material world deal with it conceptually, the essential function of concepts is just to deal with matter. Nor does such an inference respecting the essential function of concepts follow from the fact that the superior effectiveness of man's dealing with the physical world is due to his dealing with it conceptually. For it is obviously conceivable and supposable that such conceptual dealing with matter is only an incident or byplay or subordinate interest in the career of concepts. It is conceivably possible that such employment is only an avocation, more or less serious indeed and more or less advantageous, yet an avocation, and not the vocation, of intellect. Is it not evidently possible to go even further? Is it not logically possible to admit or to contend that, inasmuch as the human intellect is functionally attached to a living body which is itself plunged in a physical universe, it is absolutely necessary for the intellect to concern itself with matter in order to preserve, not indeed the animal life of man, but his intellectual life—is it not allowable, he will say, to admit or to maintain *that* and at the same time to deny that such concernment with matter is the intellect's chief or essential function and that the subjugation of matter is its ideal and aim?

Of course, our lecturer will say, our critics might be wrong in their logic and right in their opinion, just as they might be wrong in their opinion and right in their logic, for opinion is often a matter, not of logic or proof, but of temperament, taste and insight. But, he will say, if the issue as to the chief function of concepts and the ideal of the intellect is to be decided in accordance with temperament, taste and insight, then there is room for exercise of the preferential faculty, and alternatives far superior to the choice of our critics are easy enough to find. It may accord better with our insight and taste to agree with Aristotle that "It is owing," not to the necessity of maintaining animal life or the desire of subjugating matter, but "it is owing to their *wonder* that men both now begin and at first began to philosophize; they *wondered* originally at the obvious difficulties, then advanced little by little and stated the difficulties about the greater matters." The striking contrast of this with the deliverances of Bergson is not surprising, for Aristotle was a pupil of Plato and the doctrine of Bergson is that of Plato completely inverted. It may accord better with our insight and taste to agree with the great C. G. I. Jacobi, who, when he had been reproached by Fourier for not devoting his splendid genius to physical investigations, replied that a philosopher like his critic "ought to know that the unique end of science is," not public utility and applications to natural phenomena, but "is the honor of the human spirit." It may accord better with our temperament and insight to agree with the sentiment of Diotima: "I am persuaded that all men do all things, and the better they are the better they do them, in the hope," not of subjugating matter, but "in the hope of

the glorious fame of immortal virtue."

But it is unnecessary, ladies and gentlemen, it is unnecessary, our speaker will say, to bring the issue to final trial in the court of temperaments and tastes. We should have there a too easy victory. The critics are psychologists, some of them eminent psychologists. Let the issue be tried in the court of psychology, for it is there that of right it belongs. They know the fundamental and relevant facts. What is the verdict according to these? The critics know the experiments that have led to and confirmed the psychological law of Weber and Fechner and the doctrine of thresholds; they know that, in accordance with that doctrine and that law, an appropriate stimulus, no matter what the department of sense, may be finite in amount and yet too small, or finite and yet too large, to yield a sensation; they know that the difference between two stimuli appropriate to a given sense department, no matter what department, may be a finite difference and yet too small for sensibility to detect, or to work a change of sensation; they ought to know, though they seem not to have recognized, much less to have weighed, the fact that, owing to the presence of thresholds, the greatest number of distinct sensations possible in any department of sense is a *finite* number; they ought to know that the number of different departments of sense is also a *finite* number; they ought to know that, therefore, the total number of distinct or different *sensations* of which a human being is capable is a *finite* number; they ought to know, though they seem not to have recognized the fact, that, on the other hand, the world of *concepts* is of *infinite* multiplicity, that concepts, the fruit of intellect, as distinguished from sensations, the fruit of sensibility, are *infinite* in number; they ought, therefore, to see, our speaker will say, though none of them has seen, that in attemping to derive intellect out of sensibility, in attempting to show that (as James says) "concepts flow out of percepts," they are confronted with the problem of bridging the immeasurable gulf between the finite and the infinite, of showing, that is, how an infinite multiplicity can arise from one that is finite. But even if they solved that apparently insoluble problem, they would not yet be in position to affirm that the function of intellect and its concepts is, like that of sensibility, just the function of dealing with matter, as the function of teeth is biting and chewing. Far from it.

Let us have another look, the lecturer will say, at the psychological facts of the case. Owing to the presence of thresholds in every department of sense it may happen and indeed it does happen constantly, in every department, that three different amounts of stimulus of a same kind give *three sensations such that two of them are each indistinguishable from the third and yet are distinguishable from one another.* Now, for sensibility in any department of sense, two magnitudes of stimulus are unequal or are equal according as the sensations given by them are or are not distinguishable. Accordingly in the world of sensible magnitudes, in the sensible universe, in the world, that is, of *felt* weights and thrusts and pulls and pressures, of *felt* brightnesses and warmths and lengths and breadths and thicknesses and so on, in this world, which is the world of matter, *magnitudes are such that two of them may each be equal to a third without being equal to one another.* That, our speaker will say, is a most significant fact and it means that the sensible world, the world of matter, is irrational, infected with contradiction, contravening the essential laws of thought. No wonder, he will say, that old

Heraclitus declared the unaided senses "give a fraud and a lie."

Now, our speaker will ask, what has been and is the behavior of intellect in the presence of such contradiction? Observe, he will say, that it is intellect, and not sensibility, that detects the contradiction. Of the irrationality in question sensibility remains insensible. The data among which the contradiction subsists are indeed rooted in the sensible world, they inhere in the world of matter, but the contradiction itself is known only to the logical faculty called intellect. Observe also, he will say, and the observation is important, that such contradictions do not compel the intellect to any activity whatever intended to preserve the life of the living organism to which the intellect is functionally attached. That is a lesson we have from our physical kin, the beasts. What, then, *has* the intellect done because of or about the contradiction? Has it gone on all these centuries, as our critics would have us believe, trying to "think matter," as if it did not know that matter, being irrational, is not thinkable? Far from it, he will say, the intellect is no such ass.

What it has done, instead of endlessly and stupidly besieging the illogical world of sensible magnitudes with the machinery of logic, what it has done, our lecturer will say, is this: it has created for itself another world. It has not rationalized the world of sensible magnitudes. That, it knows, can not be done. It has discerned the ineradicable contradictions inherent in them, and by means of its creative power of conception it has made a new world, a world of conceptual magnitudes that, like the continua of mathematics, are so constructed by the spiritual architect and so endowed by it as to be free alike from the contradictions of the sensible world and from all thresholds that could give them birth. Indeed conception, to speak metaphorically in terms borrowed from the realm of sense, is a kind of infinite sensibility, transcending any finite distinction, difference or threshold, however minute or fine. And, now, our speaker will say, it is such magnitudes, magnitudes created by intellect and not those discovered by sense, though the two varieties are frequently not discriminated by their names, it is such conceptual magnitudes that constitute the subject-matter of science. If the magnitudes of science, apart from their rationality, often bear in conformation a kind of close resemblance to the magnitudes of sense, what is the meaning of the fact? It means, contrary to the view of Bergson but in accord with that of Poincaré, that the free creative artist, intellect, though it is not constrained, yet has chosen to be guided, in so far as its task allows, by facts of sense. Thus we have, for one example among many, conceptual space and sensible space so much alike in conformation that, though one of them is rational and the other is not, the undiscriminating hold them as the same.

And now, our lecturer will ask, for we are nearing the goal, what then *is* the motive and aim of this creative activity of the intellect? Evidently it is not to preserve and promote the life of the human body, for animals flourish without the aid of concepts and despite the contradictions in the world of sense. The aim is, he will say, to preserve and to promote the life of the intellect itself. In a realm infected with irrationality, with omnipresent contradictions of the laws of thought, intellect can not live, much less flourish; in the world of sense, it has no proper subject-matter, no home, no life. To live, to flourish, it must be able to think, to think in accordance with the laws of its being. It is stimulated and its activity sustained by two opposite

forces: discord and concord. By the one it is driven; by the other, drawn. Intellect is a perpetual suitor. The object of the suit is not the conquest of matter, it is a thing of mind, it is the music of the spirit, it is Harmonia, the beautiful daughter of the muses. The aim, the ideal, the beatitude of intellect is harmony. That is the meaning of its endless talk about compatibilities, consistencies and concords, and that is the meaning of its endless battling and circumvention and transcendence of contradiction. But what of the applications of science and public service? These are by-products of the intellect's aim and of the pursuit of its ideal. Many things it regards as worthy, high and holy—applications of science, public service, the "wonder" of Aristotle, Jacobi's "honor of the human spirit," Diotima's "glorious fame of immortal virtue"—but that which, by the law of its being, intellect seeks above all and perpetually pursues and loves, is harmony. It is for a home and a dwelling with her that intellect creates a world; and its admonition is: Seek ye first the Kingdom of Harmony, and all these things shall be added unto you.

And the ideal and admonition, thus revealed in the light of analysis, are justified of history. Inverting the order of time, we have only to contemplate the great periods in the intellectual life of Paris, Florence and Athens. If, among these mightiest contributors to the spiritual wealth of man, Athens is supreme, she is also supreme in her devotion to the intellect's ideal. It is of Athens that Euripides sings:

The sons of Erectheus, the olden,
  Whom high gods planted of yore
In an old land of heaven upholden,
  A proud land untrodden of war;
They are hungered, and lo, their desire
  With wisdom is fed as with meat;
In their skies is a shining of fire,
  A joy in the fall of their feet;
And thither with manifold dowers,
  From the north, from the hills, from the morn,
The Muses did gather their powers,
  That a child of the Nine should be born;
And Harmony, sown as the flowers,
  Grew gold in the acres of corn.[2]

And thus, ladies and gentlemen, our lecturer will say, what I wish you to see here is, that Science, and especially Mathematics, the ideal form of science, are creations of Intellect in its quest for Harmony. It is as such creations that they are to be judged and their human worth appraised. Of the applications of mathematics to engineering and of its service in natural science, I have spoken at length, he will say, in the course of previous lectures. Other great themes of our subject remain for consideration. To appraise the worth of mathematics as a discipline in the art of rigorous thinking and as a means of giving wing to the subtler imagination; to estimate and explain its value as a norm for criticism and for guidance of speculation and pioneering in fields not yet brought under the dominion of logic; to estimate its esthetic worth as showing forth in psychic light the law and order of the psychic world; to evaluate its ethical significance in rebuking by its certitude and eternality the facile skepticism that doubts all knowledge, and especially in serving as a retreat for the spirit when as at times the world of sense seems madly bent on heaping strange misfortunes up and "to and fro the chances of the years dance like an idiot in the wind"; to give a sense of its religious value in "the contemplation of ideas under the form of eternity," in disclosing a cosmos of perfect beauty and everlasting order and in presenting there, for meditation, endless consequences traversing the rational world and seeming to point to a mystical region above and beyond: these and similar themes, our speaker

[2]Translation by Professor Gilbert Murray.

will say, remain to be dealt with in subsequent lectures of the course.

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Chemistry as Affecting the Profitableness of Industry[1]

In beginning the preparation of this paper I had thought of considering chemical industry as if it were distinct from other industries, but, as the subject developed, it became very apparent that no such distinct line could be drawn. Properly speaking, all industries must be considered as chemical. It is next to impossible to imagine the existence of an industry in which chemical reactions or considerations, either directly or indirectly, do not enter. It is possible that we could define chemical industry in a somewhat restricted sense, but such a definition would hardly be other than arbitrary. The lines of demarcation would be indistinct and shadowy. The only basis for such a definition would be the attitude of the popular mind. This attitude of mind has been steadily growing towards the recognition that chemistry is an important factor in every industry, and when, in any particular case, it becomes popularly recognized that chemistry is a factor in an industry, then that industry becomes a chemical industry. Ultimately, this popular recognition will extend to all industries and the rapidity of the growth of such recognition indicates that the time is not far distant when all industries will be generally and popularly recognized as chemical.

My plan had been to discuss the profitableness of chemical industry, but if we accept this conception that all industries are chemical, it would seem better that our discussion should be broadened so as to consider the general effect of chemistry upon the profitableness of industrial operations, using the words "industrial operations" as including all phases of the actual production of wealth.

Perhaps it would be well that I should make clear the conception that all industries are chemical in one or more phases. By way of illustration, let us consider the relation of chemistry to the production of power. I think we can show that there is a very close connection between chemistry and such production, and also that there is no industry which does not depend upon the consumption of power, and if this is the case, it becomes very evident that, from the power standpoint alone, all industries are chemical industries.

Our first impressions of power are those which we ourselves are conscious of exercising, and, in practise, the simplest form of power is man power as manifested in manual labor. It is not customary, perhaps, except from the humanitarian standpoint, to consider the chemical changes in the human body, converting food into work, as factors in industry. Nevertheless, they deserve serious consideration. It is being learned daily that properly fed employees are more efficient as workmen, and the study of food problems is surely a phase of the application of chemistry to industry. In some industries, the study of the food consumed by employees has a direct bearing upon the health of the employees as affected by the industry. It is found that certain foods act as prophylactics towards certain industrial diseases, and that other foods (perhaps improperly so called) act in the opposite manner. The scientific study of foods in connection with efficient manual labor is a phase of welfare work that has not been considered to the extent it deserves. Take, on the other hand, the horse. It is true that the horse is being

[1] Chairman's address, N. Y. Section—Society of Chemical Industry, October 17, 1913.